ON THE FUNCTIONAL DEPENDENCE BETWEEN THE AGRICULTURAL YIELD AND THE FERTILIZATION WITH A SINGLE TYPE OF FERTILIZER

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Abstract: *It is common knowledge that applying fertilizers is of utmost importance for obtaining higher yields and considerable profits. People are also aware of the risk of polluting the environment if the fertilizers are not applied properly. Therefore, knowing exactly what dosage is beneficial for each crop, in the particular conditions of each region, has represented the aim of research for many years. Thus, throughout the history of research in this field, mathematicians have worked together with agrochemists in interdisciplinary teams in order to get at better results. The main objective of the present paper is to prognose unifactorially the yields for any possible dosage of fertilizer. Knowing that the models we propose represent functional relations between productions and doses applied, we also know that the best prognosis will be generated by the most accurate determination of the constants which will intervene in the function expressions. Nitrogen (* N *) - based fertilizer is known to be the most important chemical fertilizer; it is the most commonly used. Some factories produce fertilizers which contain nitrogen, phosphorus and potassium (* N, P, K *) in pre-established proportions, which for the purpose of this study can be considered as being equivalent to a single type of fertilizer. Among the first important models which give the mathematical relation between agricultural yield and fertilizers is the Mitscherlich function. Admitting that yield increase* $\frac{dx}{dy}$ *is proportional to the saturation deficit* $a-y$*, the following differential equation results:*

$$\frac{dx}{dy} = b(a-y) \quad (1)$$

the solution of which is

$$y = a\left(1 - ke^{-bx}\right), \quad (2)$$

in other words, the Mitscherlich function. In order to exemplify we apply the above-mentioned model on wheat experiences on Alex variety. The experiment was carried out at Timisoara Didactic Station between 2006 and 2009. The results are shown in Table 1. Constants a, b and k in relation (2) are determined through confrontation with experimental data, using the least squares method. When we represent graphically the Mitscherlich function and the experimental data, we get pictures 1-4. These graphics show that the experimental data are close to the theoretic curves, and this fact confirms the mathematical model.

Key words**:** *fertilization, Mitscherlich, least squares, modeling*

INTRODUCTION

Agriculture is the main source of food for people, and has been so for the most part of the history of mankind. Plant cultivation plays an extremely important role in feeding the people, but also the animals raised by people. As the population is in continuous growth, so must be the production. Thus, the areas cultivated cover more and more land, but this is not enough: the yields have to be bigger for the same patch of land cultivated with a certain type of crop. In other words, the production per individual plant must be increased. This is how fertilizers appeared; from man's strive to get as big a yield as possible from a given plot of land cultivated. From the invention of fertilization, researchers have been trying to obtain the formula for the best combination of various chemical elements, in such a way that when used in fertilization, they will give the highest yield possible.

The most important chemical fertilizer is the one based on nitrogen (*N*) as it is most commonly used. Some factories produce fertilizers with nitrogen, phosphorus and potassium (*N*, *P*, *K*) in pre-established proportions – for the purpose of this study, we will consider these as being equivalent to a single type of fertilizer.

MATERIAL AND METHODS

One of the first important models to give the mathematical link between the agricultural yield and fertilizers is the Mitscherlich function:

$$y = a(1 - ke^{-bx}) \tag{1}$$

where:

x is the quantity of fertilizer used;
y is the yield obtained;
a is the maximum yield (saturation yield).

The function in relation (1) is the solution for the differential equation:

$$\frac{dy}{dx} = b(a - y) \tag{2}$$

meaning that production increase $\left(\frac{dy}{dx}\right)$ is proportional to the saturation deficit $a - y$.

Constants a, k and b in relation (1) are determined by confrontation with the experimental data. We can apply the least squares method, which consists in the following steps.

If x_i and Y_i are the experimental data (in Table 1), we calculate the sum of distance squares, measured on ordinates, between the value of function y_i given by (1) and the experimental yield Y_i, namely:

$$F(a,k,b) = \sum_{i=1}^{n} (y_i - Y_i)^2 \tag{3}$$

which has to be minimum.

Taking (1) into consideration, relation (3) becomes:

$$F(a,k,b) = \sum_{i=1}^{n} (a - ake^{-bx_i} - Y_i)^2 \tag{4}$$

but F is minimum if:

$$\frac{\partial F}{\partial k} = 0, \ \frac{\partial F}{\partial b} = 0, \ \frac{\partial F}{\partial a} = 0 \tag{5}$$

or:

$$\begin{cases} \sum_{i=1}^{n}(a-Y_i)e^{-bx_i} - ak\sum_{i=1}^{n}e^{-2bx_i} = 0 \\ \sum_{i=1}^{n}(a-Y_i)x_ie^{-bx_i} - ak\sum_{i=1}^{n}x_ie^{-2bx_i} = 0 \\ \sum_{i=1}^{n}(a-Y_i) - ak\sum_{i=1}^{n}e^{-bx_i} = 0 \end{cases} \quad (6)$$

System (6) can also be expressed in the following way:

$$ak = \frac{\sum_{i=1}^{n}(a-Y_i)}{\sum_{i=1}^{n}e^{-bx_i}} = \frac{\sum_{i=1}^{n}(a-Y_i)e^{-bx_i}}{\sum_{i=1}^{n}e^{-2bx_i}} = \frac{\sum_{i=1}^{n}(a-Y_i)x_ie^{-bx_i}}{\sum_{i=1}^{n}x_ie^{-2bx_i}} \quad (7)$$

When solving system (6) or system (7), we find the coefficients *a*, *k* and *b*.

RESULTS AND DISCUSSIONS

To exemplify, we apply the model presented above to experiences on Alex winter wheat. The experience took place at Timişoara Didactic Station, from 2006 to 2008. The results of the research are shown in Table 1.

Table 1

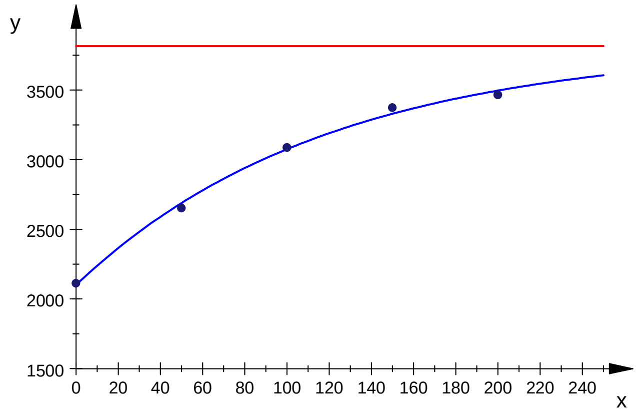
Experimental data regarding Alex wheat production, at Timişoara Didactic Station, 2006-2008

N x_i / *PK* y_i		0	50	100	150	200
P_0K_0	0	2112	2652	3087	3372	3465
$P_{50}K_{50}$	100	2449	3331	3863	4342	4724
$P_{100}K_{100}$	200	2769	2667	4395	5154	4962
$P_{150}K_{150}$	300	3002	4028	5242	5870	5893

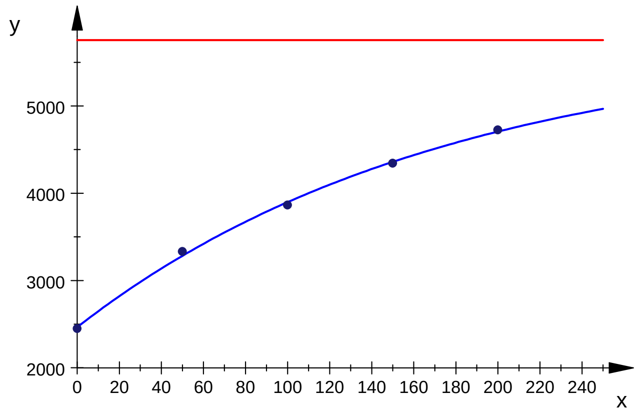
For the data in Table 1 we get the following values of the constants:

Coefficient / Variant	a	k	b
P_0K_0	3815.73	0.450	0.00841
$P_{50}K_{50}$	5754.15	0.572	0.00572
$P_{100}K_{100}$	5606.50	0.514	0.00908
$P_{150}K_{150}$	6733.74	0.565	0.00857

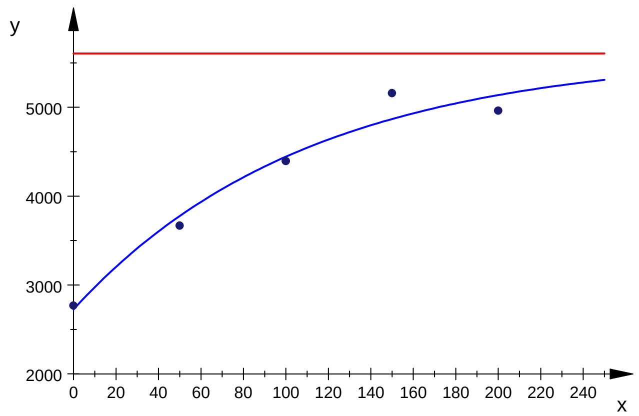
We replace the values of the constants in relation (1) and we represent graphically both function (1) and the experimental data in Table 1.



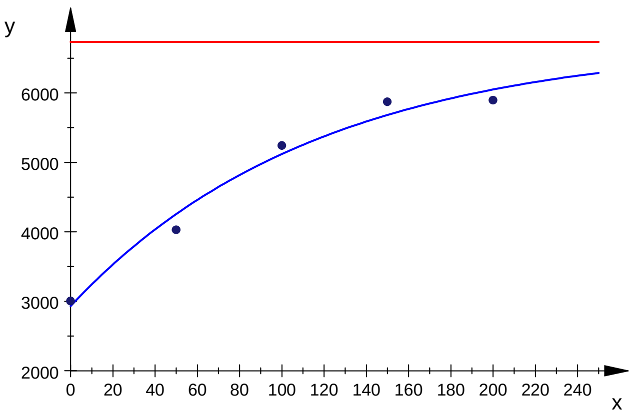
Picture 1. Wheat production in relation to nitrogen-based fertilizers, variant P_0K_0



Picture 2. Wheat production in relation to nitrogen-based fertilizers, variant $P_{50}K_{50}$



Picture 3. Wheat production in relation to nitrogen-based fertilizers, variant $P_{100}K_{100}$



Picture 4. Wheat production in relation to nitrogen-based fertilizers, variant $P_{150}K_{150}$

The graphics clearly show that the experimental data are very close to the theoretic curves, fact which confirms the mathematical model we proposed.

CONCLUSIONS

Our research showed the differentiated influence of various types and doses of fertilizers used for winter wheat fertilization in the conditions at Timişoara Didactic Station.

Fertilizers with phosphorus and potassium led to yields of 337 kg/ha for variant $P_{50}K_{50}N_0$, 657 kg/for variant $P_{100}K_{100}N_0$ and 890 kg/ha for variant $P_{150}K_{150}N_0$.

When considered by itself, nitrogen led to yields between 540 and 1353 kg/ha. When we analyze the complex fertilizer as a whole, as a single type of fertilizer, the effect is different, because it is amplified by the synergic effect of the nutritious elements in its composition. Thus, the production increase ranges between 1219 and 3781 kg/ha.

If we deal with this aspect mathematically, namely through modeling, the functional dependence between the agricultural yield and fertilizers in the case of one-fertilizer fertilization is given by Mitscherlich function.

The constants which intervene in the expression of the above-mentioned function can be determined by using the least squares method in confrontation with the experimental data.

Pictures 1 to 4 show good consistency between the experimental data and the theoretic curves.

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