MATHEMATICAL MODEL CONSTRUCTION OF THE ISOTROPIC FILTRATION PROCESS BASED ON DARCY'S LAW

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Abstract. *Filtration is one of the main impacts on hydraulic structures. This phenomenon causes consequences such as loss of water from reservoirs, force impact on the structure in the form of filtration back pressure or volumetric hydrodynamic filtration forces. In practice, isotropic filtration is more often considered, characterized by the same conductivity of materials in all directions [1]:*
$K_x = K_y$. *Under the influence of the pressure created by the dam, water is filtered through the body of the dam from the upstream to the downstream. The upper limit of the filtering will be a line called the depression curve. If the depression curve tapers out on the downstream slope of the dam, then there is water filtration under some residual pressure. In this case, the wedged out water begins to flow in streams down the slope surface. The water wedged out on the slope washes soil particles out of the body of the dam. This disturbance of the stability of soil particles, called suffusion, leads to increased filtration, and then to slumping and failure of the downstream slope, which poses a direct threat to the entire dam. The article considers the review problems of mathematical modeling of plane-parallel isotropic fluid filtration.*

Keywords*: hydraulic engineering construction, mathematical modeling, plane-parallel filtration, incompressible fluid, isotropic, earth dam, homogeneous.*

INTRODUCTION

Earth dams are widely used in the practice of hydraulic construction due to the simplicity of design, durability and low cost of construction. In particular, they are designed to concentrate pressure at the location of the structure, to create reservoirs, provide navigable depths, irrigation, land watering, regulation of river water flow for water supply, i.e. for economic, energy and recreational purposes [2].

According to the design of the transverse profile, the simplest is a jumper with vertical slopes, made of homogeneous soil. The best materials for homogeneous earth dams are loam, sandy loam, sandy and sandy-gravelly soils [3].

The movement of a liquid (gas, carbonated liquid) in a porous medium is called filtration [4].

Filtration theory studies the flow (leakage) of liquids and gases through porous media. The porous medium is a solid skeleton penetrated by a complex system of channels and cracks. It can also be considered as a set of solid particles (grains) closely adjacent to each other [5].

MATERIAL AND METHODS

In most cases, porous media have an irregular internal structure. This circumstance complicates a detailed description of filtration flows by direct methods of hydrodynamics, which involve solving the equations of motion of a viscous fluid in the region of the pore space. An approach appears to be effective in which the porous skeleton and the fluid filling it are considered as a continuous medium.

The main characteristics of such a medium (pressure, density, speed) at each point in space are determined by averaging over some area containing this point. In this case, the dimensions of the averaging region should be significantly smaller than the characteristic dimensions of the porous layer, on the one hand, and sufficiently large compared to the sizes of pores or grains, on the other.

One of the main characteristics of the selected area of a porous medium is porosity – the ratio of the volume of the area occupied by pores to its total volume. Thus, if in the volume V of a porous material, the pores account for the volume V_p, then the porosity of such a material will be:

$$m = \frac{V_p}{V} .$$

Porosity limits the amount of fluid that saturates the porous medium. In an inhomogeneous medium, the porosity at a given point *M (x, y, z)* of space is determined by the limit:

$$m(M) = \lim_{\Delta V \to 0} \frac{\Delta V_p}{\Delta V} = \frac{dV_p}{dV} .$$

Porosity is a dimensionless quantity with values in the range $0<m<1$, which characterizes the shape and mutual arrangement of grains (pores) and therefore is the same for geometrically similar media. Soil porosity lies in the range 0.3÷0.7, wood – 0.45÷0.7, oil and gas bearing formations – 0.1÷0.2. For a more complete description of a porous medium, it is also necessary to indicate the characteristic size of the pore space, i.e., the average size of the pore channel or grain of the porous skeleton.

There are materials whose total porosity is different from zero (that is, pores exist), but the active one is zero – all pores are isolated and filtration is impossible. An example of such material is cork tree bark.

Along with porosity, such a characteristic of porous media as transparency or surface porosity is singled out. To determine it through an arbitrary point of the porous medium in a certain direction, a section is made by a plane.

Let the area of the obtained section S, and the area of its part attributable to the pores – S_P. The ratio of the pore area to the total cross-sectional area is called transparency:

$$n = \frac{S_p}{S} .$$

For inhomogeneous media, the value of transparency at a given point M(x, y, z) of the space is determined by the limit:

$$m(M, \mathbf{n}) = \lim_{\Delta S \to 0} \frac{\Delta S_p}{\Delta S} = \frac{dS_p}{dS} ,$$

Where, n is the normal vector to the section plane.

The concepts of porosity and transparency in the general case are not identical. Transparency significantly depends on the direction of the section. However, in many porous media, the average value of the transparency (for sections in different directions) coincides with the porosity, that is, $m=n$. This property underlies one of the methods for measuring porosity.

Another important characteristic of a porous medium is the specific surface of the pore space, which is equal to the ratio of the surface area of the part of the volume occupied by pores S_p to the entire volume V:

$$\Sigma = \frac{S_p}{V} .$$

Unlike transparency and porosity, the specific surface of the pore space is a dimensional value.

The main characteristic of filtration is the filtration speed *u*. In the general case of a non-one-dimensional flow, the vector quantity *u* is defined as follows. An elementary area *ΔS* with normal *n* is allocated around some point of the porous medium. Then the projection of the filtration speed vector onto the normal *n* at this point is equal to the limit of the ratio of the volume flow rate of the liquid *ΔQ* through the platform to its area *ΔS* [5,6]:

$$u_n = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S}$$

It is important to emphasize that the volume flow here is divided by the entire area ΔS, and not by the area occupied by the pores, so the filtration speed does not coincide with the true speed of individual fluid particles, but approximately *m* (*m* – porosity) times less.

The proof is easy to carry out for one-dimensional filtration of a homogeneous incompressible fluid. The constancy of the flow in this case leads to the following equalities:

$$Q = u \cdot S = v_{av} \cdot S_p ,$$

where v_{av} is the average speed of liquid particles, determined by the cross-sectional area occupied by pores.

And since the clearance n is approximately equal to the porosity *m*, in the end we get:

$$u = m \cdot v_{av}.$$

In hydraulic engineering, when considering filtration, we mean porous media formed from soils (cohesive and non-cohesive), fractured rocks, concrete and other porous materials, and water is considered as a filtering liquid [6].

The pores of soil, concrete, cracks in rocks, in which fluid moves, have complex and diverse shapes. This circumstance leads to a special method for studying the motion of a fluid in a porous medium. It is precisely because of the absence of regularities in the shapes of pores and cracks that the averaged characteristics of the filtration properties of a porous medium are considered [6].

A change in the mass of liquid in an arbitrary fixed volume *V* can occur due to the influx of liquid through the boundary ∑, which limits this volume:

$$\frac{d}{dt}\int_V \rho m dV = -\int_\Sigma \rho u \cdot n d\sigma ,$$

Where, *n* is the outward normal vector to the surface ∑.

The surface integral on the right side of the last equality can be transformed using the Gauss–Ostrogradsky formula. The derivative on the left side (for sufficiently smooth functions) is brought under the integral sign:

$$\int_V \frac{\partial(\rho m)}{\partial t} dV = -\int_V div(\rho u) d\sigma$$

Finally, the differential form of the law of conservation of mass (or the equation of continuity) is obtained due to the arbitrariness of the volume *V*:

$$\frac{\partial(\rho m)}{\partial t} + div(\rho u) = 0$$

RESULTS AND DISCUSSIONS

The momentum balance equation (Darcy's law) was obtained experimentally by the French hydraulician Henri Darcy – water was passed through a vertical column of sand (Fig. 1). Defining pressure as

$$H = \frac{p}{\rho g} + z,$$

where p is the pressure, g is the free fall acceleration, z is the vertical coordinate, Darcy found that the fluid flow rate Q is directly proportional to the difference in pressure in the upper and lower sections of the column, the section area S and inversely proportional to the distance between the sections $L=z_1-z_2$:

$$Q = k_f \frac{H_1 - H_2}{L} S \ ,$$

where k_f is the filtration coefficient, which depends on the properties of the liquid (primarily the dynamic viscosity μ) and the porous skeleton.

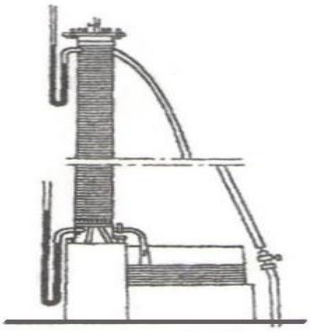

Fig. 1. Scheme of the Darcy experimental setup for studying the flow of water through sand

The last equation can be rewritten as:

$$Q = \frac{k}{\mu} \frac{p_1 * - p_2 *}{L} S,$$

where $p^*=p+\rho gz$ – the reduced pressure, $k=k_f\mu/(\rho g)$ – the permeability coefficient, which now depends only on the properties of the porous skeleton.

The permeability coefficient has the dimension of area. The unit of measure Darcy (D) turns out to be convenient in practice: 1D $=1.02\cdot 10^{-12}$ m^2 . Permeability of oil-bearing layers ranges from 10^{-3}D to 1D.

Using the definition of filtration rate, you can get:

$$u = \frac{k}{\mu} \frac{p_1 * - p_2 *}{L}.$$

The passage to the limit as L0 leads to the differential form of Darcy's law for an isotropic porous medium:

$$u = -\frac{k}{\mu} \operatorname{grad} p * \qquad (2)$$

The minus sign indicates that the velocity is directed in the direction opposite to the increase in pressure.

Let a homogeneous incompressible fluid with a dynamic viscosity μ be filtered through a rectangular isotropic region with a permeability k , so that the streamlines are parallel to the OX axis. The flows in Darcy's experimental setup approximately correspond to these conditions [4-10].

Upper $y = l$ and lower $y = 0$ boundaries of the region are impenetrable. At the input of the region $x = 0$, a constant pressure $p = p_1$ is set, at the output $x = L$ – a constant pressure $p = p_2 < p_1$. There are no mass forces.

The continuity equation and Darcy's law are reduced in this case to the system:

$$\begin{cases} \dfrac{du_x}{dx} + \dfrac{du_y}{dy} + \dfrac{du_z}{dz} = 0 \\ u_x = -\dfrac{k}{\mu}\dfrac{dp}{dx}, u_y = u_z = 0 \end{cases}$$

From which the only unknown speed component u_x is easily excluded:

$$\frac{d^2 p}{dx^2} = 0$$

Integrating the last equation we get:

$$\frac{dp}{dx} = C_1$$

Where,

$$p = C_1 x + C_2$$

From the boundary conditions ($x = 0$: $p = p_1$; $x = L$: $p = p_2$) we find the integration constants:

$$C_1 = \frac{p_2 - p_1}{L}, C_2 = p_1$$

As a result, we obtain that in the case of a rectilinearly parallel filtration flow, the pressure is distributed as follows:

$$p(x) = p_1 + \frac{p_2 - p_1}{L} x$$

The velocity component u_x is determined from the pressure distribution:

$$u_x = -\frac{k}{\mu}\frac{dp}{dx} = -\frac{k}{\mu}C_1 = \frac{k}{\mu}\frac{p_1 - p_2}{L}$$

In practice, colored particles or radioactive isotopes are often added to the seepage stream to track their movement.

The average true speed of such a «tagged particle» in the case of an isotropic medium is determined by the relation:

$$v = \frac{dx}{dt} = \frac{u_x}{m}$$

Where, m is porosity. From the last equality we get:

$$dt = \frac{m}{u} dx$$

whence it is easy to find the time during which the “tagged particle” travels the distance from the origin to an arbitrary point with coordinate x in homogeneous porous medium:

$$t = m\int_0^L \frac{dx}{u(x)}$$

Thus, for a plane-parallel fluid flow in a homogeneous porous medium, the average transit time of a «marked particle» through a rectangular region of length L is:

$$T = m\int_0^L \frac{dx}{u_x} = \frac{\mu m}{k}\frac{L^2}{p_1 - p_2}$$

CONCLUSIONS

Filtration in hydraulic structures causes consequences such as loss of water from reservoirs, as well as the force effect on the structure or volumetric hydrodynamic filtration forces. Therefore, taking into account filtration equally affects the strength properties of all materials, and thereby the ecology of the environment.

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