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GEOGEBRA 3D CALCULATOR AS A TOOL FOR INTERACTIVE LEARNING METHOD IN GEODETIC ENGINEERING

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Abstract. *In this paper we emphasize the usefulness of using GeoGebra and specifically GeoGebra 3D Calculator in the educational process, for first year students, Land Surveying and Cadastre, at the Faculty of Geodesy of the Technical University of Civil Engineering Bucharest. The implementation of this way of teaching in the course Geometric Representations of Topographic Surfaces was initiated two years ago, in a gradual way, in order to support the students to understand well certain concepts. We started with a bibliometric study on this software product, then we showed how we implemented its use in the applied training of students. We have developed as training materials an e-book and a book in classical format, with problems solved both in the classical way and as examples using the software. These approaches have led to better results in the learning process, as well as to a more active involvement of the students during the application classes. Supporting classroom problem-solving with experienced applications significantly improves students' problem understanding. GeoGebra fosters a collaborative learning environment through group investigations, conjecture-sharing, and alternative solution discussions. This helps students develop essential critical thinking, communication, and teamwork skills. Additionally, GeoGebra aids formative assessment; interactive applets allow educators to track students' reasoning processes. Experience in the Geometric Representations course shows GeoGebra motivates learning mathematics overall and reduces anxiety for first-year students. Students became more self-critical of their work, questioning their errors in problem-solving and gaining the opportunity to visually identify those mistakes.*

***Keywords**: GeoGebra 3D Calculator; augmented reality; technology enhanced learning; Geodetic Engineering; dynamic geometric software; open source software; 3D visualization*

INTRODUCTION

In general, underachievement in spatial reprsentation from 3D to 2D is a real, complex and urgent problem that requires effective solutions, and one of those solutions is the use of apps to aid the understanding process.

GeoGebra is a software widely used in the educational process in universities, being used for solving geometry problems, but also for some GIS applications, especially those related to the mathematical basis of cartographic projections.

The digital age presents both relevant challenges and substantial opportunities for mathematics education, demanding a pivot toward more student-centric and interactive learning methodologies. Integrating digital instruments into the classroom has become indispensable; these technologies facilitate dynamic visualization, enhance the grasp of abstract concepts, and elevate student engagement. (SIREGAR, 2025)

Research highlights that digital-based learning environments cultivate deeper comprehension and creative problem-solving by enabling students to engage in mathematical exploration through manipulation and experimentation. Moreover, the recent expansion of online and hybrid learning models has amplified the imperative to develop pedagogical frameworks that effectively utilize digital platforms, such as GeoGebra, to maximize the accessibility and impact of mathematics instruction. GeoGebra is widely recognized as a premier software solution designed to meet the modern directive for visualization techniques in mathematics, adeptly building geometric constructions.

Spatial 3D visualization is the ability to spatially combine all details related to the shape of geometric objects and their position. Students have to solve the applications, and as a necessary flow the process of mentally rotating the geometric bodies in the projection planes has to be taken into account, so three skills are required: the ability to mentally rotate the geometric body in the projection plane, the ability to visualize in detail the shape of the geometric body and the ability to realize the relationship between the features that will interact with the geometric body. (CHIVAI et.al., 2022)

GEOGEBRA AS DIGITAL TEACHING TOOL

According to Yohannes and Chen, 2021, it was found that there have been a limited number of studies on integrating GeoGebra into mathematics education. It was also shown that most of the studies adopted activity/task-based learning strategies to implement GeoGebra.

As stated in Aguirre-Molina and Gras-Velázquez, 2011, the success of a project, called SPICE, which included 16 participating countries, was based on the collaboration between a panel of teachers and a panel of science experts to find the best practice experiences. As a result 24 projects were selected and implemented in the educational process, and one of them was based on GeoGebra and had the title "Earth radius (Eratosthenes's method) using Geogebra".

In the literature it has been highlighted the knowledge and emergent mathematical work of high school students when solving homework tasks using GeoGebra. (TICSE AUCAHUASI ET AL., 2023) The acquired knowledge of Chilean students was analyzed and it was concluded that their mathematical understanding was improved. The effects of using a dynamic geometry software in active learning framework has been discussed and the main conclusion was that GeoGebra is perceived predominantly as Facilitates Learning, then as Quick Perception and as Motivator. (CHIVAI ET.AL., 2022) It is concluded that GeoGebra with AR (GeoGebra 3D Calculator) becomes a valuable tool that allow students to interact with three-dimensional models, graphics, and other visualizations so that they can understand mathematical concepts more effectively. (IPARRAGUIRRE-VILLANUEVA, 2024)

Comparative studies were also carried out involving teachers who teach mathematical knowledge to construction students, and the conclusion was that construction students may perform better in terms of demonstrating mathematical competence and applying mathematical concepts in construction when engaged in technological design activities. (SAPARBAYEVA, 2024)

The use of GeoGebra was also studied as an effect on teacher training, the analysis carried out through the organization of a mathematics training workshop using GeoGebra software was studied on changes in teachers' knowledge and beliefs (ALIZADEH-JAMAL ET. AL., 2018). For university-level courses, GeoGebra acts as a critical interface between abstract mathematical theory and practical scientific application. It empowers students across STEM fields, including engineering, physics, and economics, to conduct precise simulations of real-world scenarios and clearly visualize mathematical interdependencies, thereby strengthening interdisciplinary understanding. (SIREGAR, 2025)

The focus on digital tools was highlighted by articles showing the importance of modern methods like e-learning platforms. (Badea et al., 2012) In another paper, the authors emphasized the applicative innovative teaching modules in higher education – land surveying and cadastral domain. (BADEA, BADEA, 2014)

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GEOGEBRA – A SHORT BIBLIOMETRIC STUDY

GeoGebra originated in Austria with Markus Hohenwarter, who developed it as part of his master's thesis (2001/2002) and PhD research at the University of Salzburg. The software quickly gained international recognition and numerous awards, being translated by educators worldwide into over 25 languages. Since 2006, the Austrian Ministry of Education has helped ensure the software remains free for educational institutions. GeoGebra's development expanded to the United States in July 2006 at Florida Atlantic University, supported by the NSF project Standard Mapped Graduate Education and Mentoring.

There are still a number of articles with this topic, most of them showing applied experiences following the implementation of this interactive way of teaching, a search in the Web of Science Core Collection shows 382 results based on the keywords GeoGebra and education. An exploratory study is seeking to confirm that GeoGebra use has increased in importance over time. A search on the keyword GeoGebra resulted in 1039 articles indexed in the WoS database. Figures 1 and 2 highlight the correlation as network and overlay based on these resources.

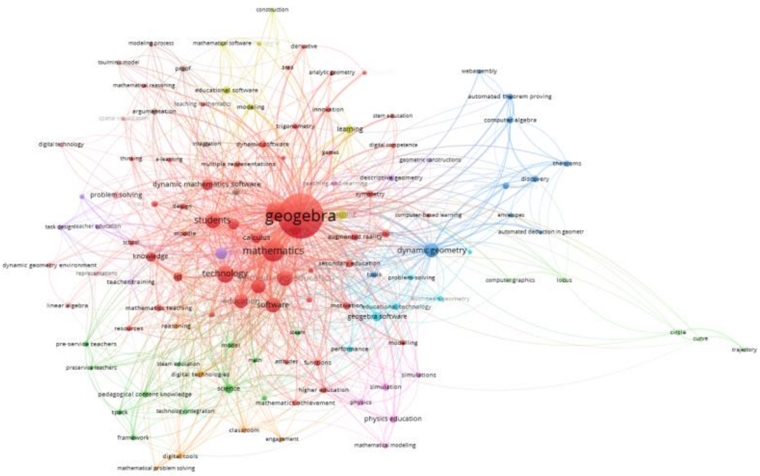

Figure 1. Network Visualization of co-occurrence of the keyword GeoGebra

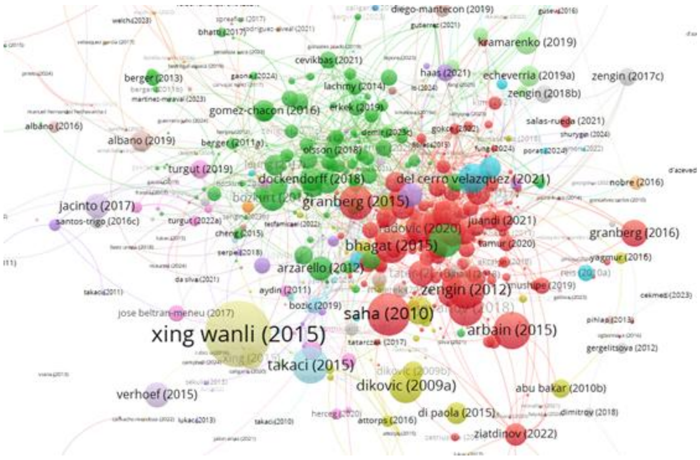

Figure 2. Overlay Visualization of the Bibliographic Coupling using keyword GeoGebra

In figure 3 we identified the network visualization of the bibliographic coupling using keywords GeoGebra and Education (using Fractional Counting), while in figure 4 there is the network visualization on the same keywords.

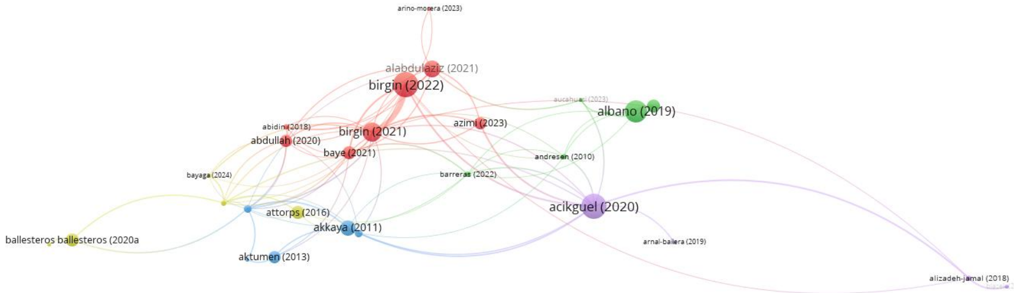

Figure 3. Network Visualization of the Bibliographic Coupling using keywords GeoGebra and Education (using Fractional Counting)

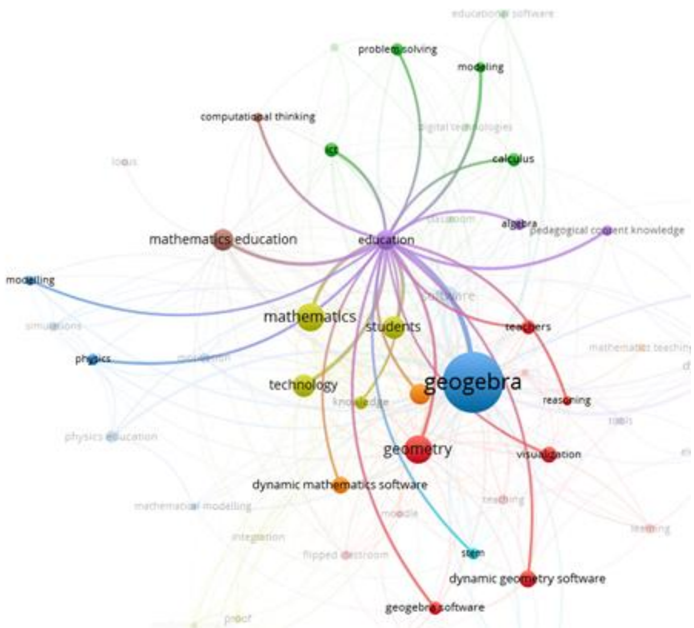

Figure 4. Network Visualization highlighting the co-occurence using keywords GeoGebra and Education (using Fractional Counting)

THE PROPOSED WORKFLOW

In 2022 we began gradually integrating GeoGebra into the learning process to help students with spatial visualization challenges interact with 3D objects in a real-world environment. By 2024 we have developed a series of applications, integrated in print and as an e-book. (BADEA ET. AL., 2024) We have included in this volume the QR code for each problem, it is possible to access the solution in 3D and then with AR it can be visualized it in the real world.

These resources were used for first year students, 121 in number.

GeoGebra's interconnected views are the following:

1. Algebraic View - to display the formal definition and data for every object in the construction.
2. Geometric View (Graphics View) - the central visual and dynamic workspace for construction.
3. Spreadsheet View - to organize and process data sets using a familiar cell-based structure.
4. CAS View (Computer Algebra System) - to perform exact, symbolic calculations.
5. Construction Protocol View - to provide a step-by-step logical history of the project.
6. Command Line - to enable precise, advanced object creation and manipulation via text input.

All visualization methods offered by this software were used to train students. The most vital feature is that all six views are dynamically linked. When an object is altered in one view—such as dragging a point in the Geometric View—its corresponding data is instantly updated in the Algebraic View, Spreadsheet View, and CAS View. This continuous synchronization offers a profound and immediate way to explore the fundamental interrelationship between algebraic, geometric, and numeric representations.

The GeoGebra 3D application was used to support the visualization process in space and to help them interact with objects, position them on a surface and visualize them from different angles.

It is also possible to access the applications from the browser, on a desktop system, but also using a smartphone. (figure 5) As main steps for the learning process, we designed a workflow to ease the learning process.

The student starts by scanning the QR code in the book for the application to be solved, visualizes the situation in space, uses augmented reality to interact with the elements in the representation and then can solve the given problem much easier. (figure 6)

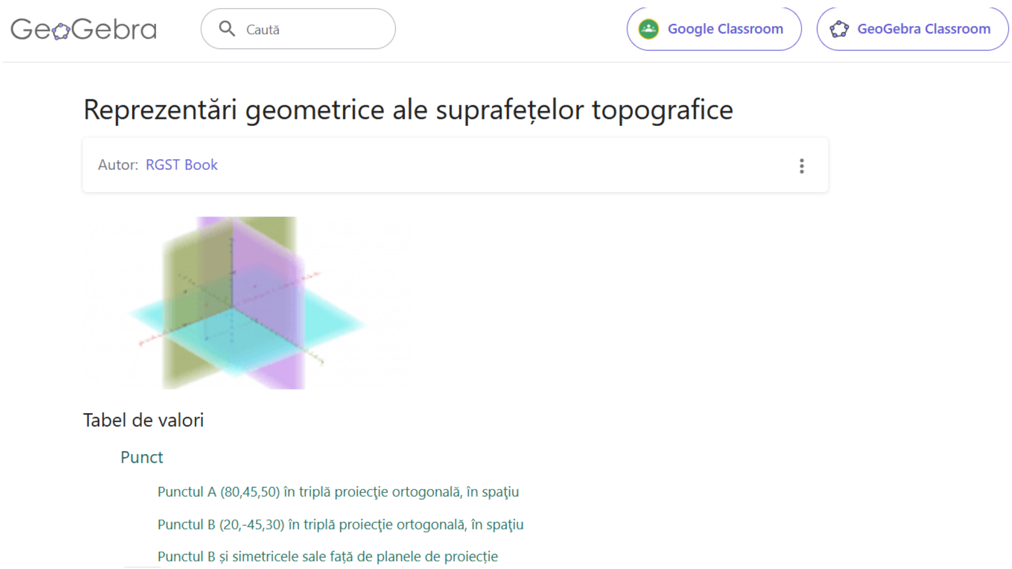

Figure 5. GeoGebra Page for accessing the applications (https://www.geogebra.org/m/dfxcqaju)

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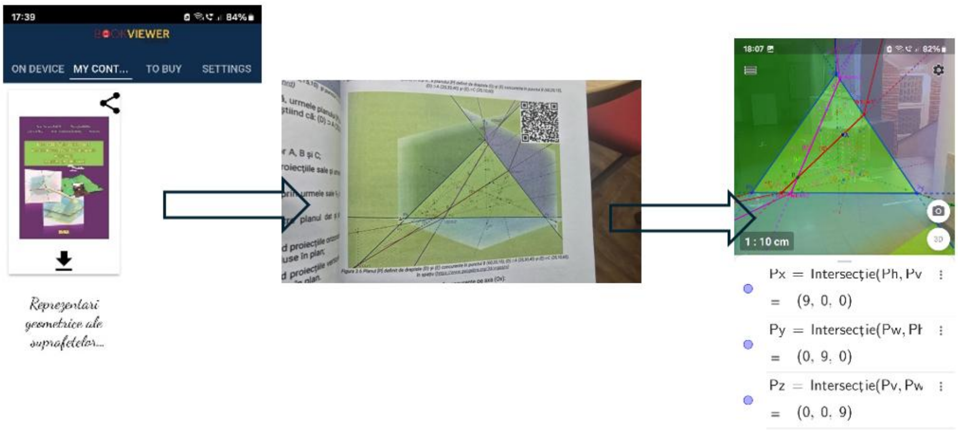

Figure 6. The proposed workflow for learning – an example

DEM bilinear interpolation is another important problem, and it can be solved like in figure 7, by designing four sliders for the elevation of the points.

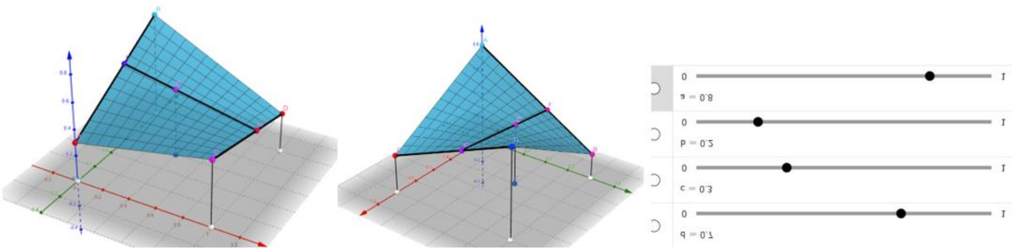

Figure 7. DEM bilinear interpolation and the sliders

In geodetic engineering, the AR tool accessible through the app installed on the phone proved to be very useful for students, giving them the opportunity to interact with the 3D objects they design. The core benefit of GeoGebra Augmented Reality (AR) is its power to highlight complex geometric abstractions. Traditional methods, relying on mere symbols or flat, 2D graphics, struggle to convey concepts like solid geometry, 3D vectors, parametric functions, and quadratic surfaces. GeoGebra AR transcends this limitation, not only providing visualization but also enabling direct, interactive manipulation of these mathematical objects, fostering an experience-driven, exploratory learning paradigm.

This is an active engagement which also serves to significantly sharpen mathematical problem-solving skills, by allowing students to physically adjust and experiment with mathematical constructs, GeoGebra AR facilitates the visual testing of hypotheses, encourages the exploration of multiple solution pathways, and cultivates a deeper, more intuitive grasp of the interrelations between mathematical components. This approach directly embodies the tenets of constructivist learning, asserting that knowledge acquisition is optimized when learners actively build understanding through hands-on interaction and discovery. (Gusteti et al, 2025)

While a broad base of research confirms that Augmented Reality technologies elevate conceptual understanding and motivation in mathematics, the specialized literature on GeoGebra AR's specific efficacy in improving problem-solving abilities remains underdeveloped. When students manipulate a 3D geometric model using the GeoGebra AR application on a tablet, they are engaging in an experience-driven and exploration-based learning approach that directly addresses the difficulty of grasping abstract concepts in geometry and graphic representations, as well as understanding the link between 2D and 3D. (figure 8)

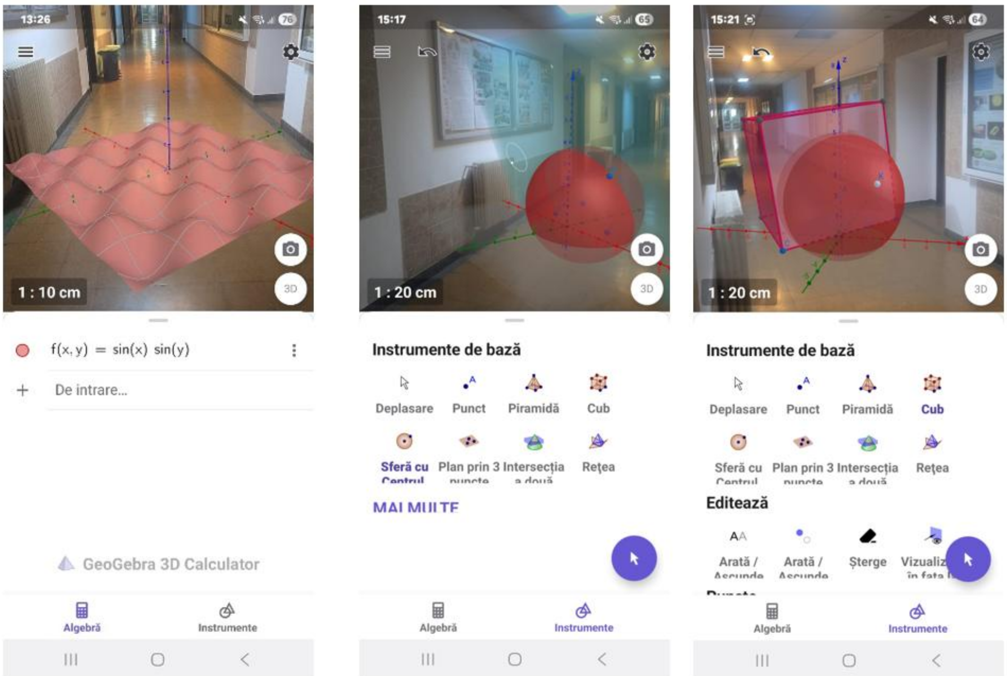

Figure 8. Interacting with 3D solids in AR environment

RESULTS AND DISCUSSION

Using GeoGebra in teaching process has demonstrated significant pedagogical advantages. Findings indicate that this dynamic software enhances students' conceptual understanding and visualization skills while simultaneously increasing their engagement with mathematical concepts used in engineering.

Empirical evidence from classroom observations revealed a direct link between students' motivation and the ability to dynamically manipulate and visualize mathematical objects.

The interactive approach available in GeoGebra is particularly effective in enabling students to explore the critical connection between algebraic and geometric forms, thereby fostering profound comprehension of abstract ideas.

These findings align with existing research, consistently showing that augmented reality (AR) significantly benefits scientific subjects requiring critical analysis. Specifically, we emphasize that students demonstrate improved conceptual learning in AR environments, as the technology makes complex information and otherwise imperceptible real-world situations easier to grasp. Augmented reality visuals played a pivotal role in enhancing their understanding, ultimately driving their motivation to learn. Collectively, these factors make the augmented reality environment a preferred choice for the teaching as well as for the learning process.

CONCLUSIONS

The experience of teaching in the course Geometric Representations of Topographic Surfaces shows that GeoGebra is motivating for learning mathematics in general and can reduce first-year students' anxiety about the subject. Students questioned the errors they made for the problems they solved and had the opportunity to see these errors.

Therefore, supporting problem-solving activities with this type of experienced applications in the classroom can make significant contributions to students in terms of improving their understanding of the problems solved.

GeoGebra fosters a collaborative learning environment, by engaging students in group investigations, conjecture-sharing, and discussions of alternative solutions, the software contributes to the development of essential critical thinking, communication, and teamwork skills. In addition, GeoGebra proves valuable for formative assessment, allowing educators to track students' reasoning processes using interactive applets.

The students' perspective was positive towards solving problems using GeoGebra, so implicitly this way of working should be included in the applications classes of this discipline. Based on the use of the GeoGebra 3D calculator, application classes on site become more dynamic and engaging, motivating students but also helping them to understand complex problems at home.

To ensure the successful integration of GeoGebra, teachers must receive appropriate training, and educational institutions are urged to provide essential technological support. Additionally, future research should investigate the long-term effects of GeoGebra instruction on students' mathematical reasoning and motivation across different academic stages. As a future direction, we could seek to fill that empirical gap by rigorously comparing the impact of GeoGebra AR against conventional instructional techniques on students' mathematical problem-solving outcomes.

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