COMPARATIVE ANALYSIS OF MERCATOR AND U.T.M. MAP PROJECTIONS

R. BERTICI*, M. HERBEI*, Silvica ONCIA*, Laura SMULEAC*

[1]BUASMV „King Michael I of Romania" from Timisoara
mihai_herbei@yahoo.com

Abstract. *The analyzed projections from this paper are cartographical projections of great importance in practice, solving cartographical problems regarding map drawing based on practical needs. These are the ones referring to terrain surfaces that will be represented, to the accuracy that must be obtained, as well as the ways to accomplish favorable and precise links with other kinds of cartographical projections.* Cylindrical Projection (Mercator): *is based on a cylinder tangent to the equator. Good for equatorial regions but greatly distorted at high latitudes. This one of the oldest and most common projectionsThe maps in Mercator projection have a great importance in maritime and air navigation, due to the fact that it is a conform projection and the cartographic network formed in perpendicular lines, the loxodrome will be a straight line. The same line makes with each projection of the meridians the same azimuth. The UTM system that uses the Mercator projection can be used all over the world having the advantage that it reduces the errors of representation in plan due to introducing a scale factor that makes that the linear distortions from the margin of the spindle projected in plan to reduce to half. In this projection it is impossible to represent whole surface of the earth on the same plan, the projection being made on different plans, each of them along one meridian called center meridian. The UTM projection is used especially in military activities. UTM is a commonly used projection for USGS maps ranging in scale from 1:24,000 to 1:250,000. The UTM projections are based on 60 UTM Zones each defined by a central meridian and covering 3 degrees of Longitude to the East and West. Maps based on the UTM Projection have a Cartesian Coordinate grid system which is used to define any point on the map. Positions are defined by the UTM Zone, an 'X' coordinate called the Easting (in meters) and a 'Y' coordinate called the Northing (in meters).*

Key words: Mercator, Map projection, distortion, cartography

INTRODUCTION

The analyzed projections from this paper are cartographical projections of great importance in practice, solving cartographical problems regarding map drawing based on practical needs. These are the ones referring to terrain surfaces that will be represented, to the accuracy that must be obtained, as well as the ways to accomplish favorable and precise links with other kinds of cartographical projections.

The maps in Mercator projection have a great importance in maritime and air navigation, due to the fact that it is a conform projection and the cartographic network formed in perpendicular lines, the loxodrome will be a straight line. The same line makes with each projection of the meridians the same azimuth.

The UTM system that uses the Mercator projection can be used all over the world having the advantage that it reduces the errors of representation in plan due to introducing a scale factor that makes that the linear distortions from the margin of the spindle projected in plan to reduce to half. In this projection it is impossible to represent whole surface of the earth on the same plan, the projection being made on different plans, each of them along one meridian called center meridian. The UTM projection is used especially in military activities.

MATHERIAL AND METHODS

- **CYLINDRICAL RIGHT CONFORMAL PROJECTION WITH INCREASING LATITUDES - MERCATOR**

This projection was established in the year 1569, by the Dutch cartographer Gerhard Kremer as know as Mercator. The Mercator projection consists in the representation of the globe on a spread surface of a cylinder whose ax coincides with the rotation ax of the Earth and it also is tangent along the Equator or secant to any two parallels (so it is a straight cylindrical projection). The Mercator projection is nothing else but the transformed straight square cylindrical projection from arbitrary (equidistant) to conformal. The distance between parallels in this projection has been elongated in the same proportion as the distance between meridians. On the world map built in this projection the surfaces of the land areas located on large latitudes are very deformed.

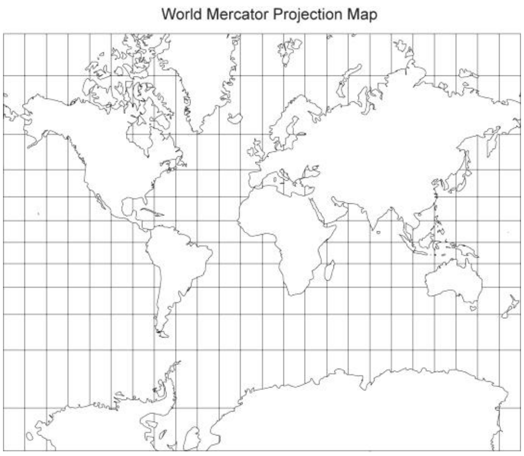

Fig. 1. The world map in Mercator projection

The cartographical network has the aspect of some rectangles elongates more and more on the meridians as the latitude increases. This is the reason why it is also called the projection with increasing latitudes.

The cartographical network is built in a practical way until $\pm 80^0$ latitude because the poles cannot be represented. It is a conformal cartographical projection which deforms very much the surfaces. From this reason it is not recommended for building educational maps because it would create a false image of the land and water.

On the direction of the meridians there are no distortions, because:

$$m = b = 1$$

$$v = b - 1 = 1 - 1 = 0 \qquad (1)$$

On the direction of the parallels the distortions took the form of elongation, because:

$$n = a = \sec\varphi \qquad (2)$$

but:

$$1 \prec \sec\varphi \prec \infty$$

so:

$$v = a - 1 \succ 1 \qquad (3)$$

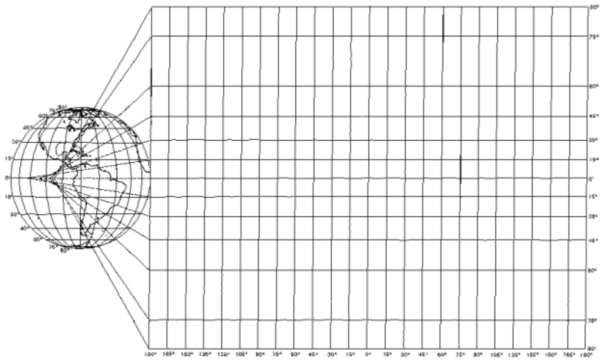

Fig. 2. The normal network in Mercator projection

In Mercator projection, we have: n = sec φ, m = sec φ and as result: m = n = sec φ or a = b = sec φ. It means that in this projection the **linear distortions** are equal in any directions with sec φ. But the **distortion ellipse** is the geometrical place of all the vector extremities which have as measure the values of the linear distortion modules. Because these vectors are equal, we no longer have a distortion ellipse, but a circle with r = sec φ.

The values of linear distortion will be:

$$v = C - 1$$

$$v = \sec\varphi - 1 \tag{4}$$

sec φ varies from 1 to ∞, so in this projection the linear distortion will have the aspect of elongations. These distortions are growing along with the latitude.

The polar regions cannot be represented, because:

$$\sec 90^0 = \infty \Rightarrow v = \infty - 1 = \infty \tag{5}$$

The distortion of direction is revealed by the following formula:

$$\sin\omega = \frac{a-b}{a+b} \tag{6}$$

Because a = b , then:

$$\sin\omega = \frac{a-b}{a+b} = \frac{0}{2a} = 0 \tag{7}$$

The angular distortions are null:

$$2\omega = 2\cdot 0 = 0 \tag{8}$$

In conclusion, the Mercator projection is a conformal projection, because the directions, as well as the angles are not distorted and as result, a circle on the sphere will be represented on the projection again like a circle, which means that the distortion ellipses will be circles, but having different surfaces.

Table 1.

The distortion on Mercator projection

φ	m=n	p	2ω
90	∞	∞	000’
80	5.759	33.166	000’
70	2.924	8.550	000’
60	2.000	4.000	000’
50	1.556	2.421	000’
40	1.305	1.703	000’
30	1.155	1.333	000’
20	1.064	1.132	000’
10	1.015	1.030	000’
0	1.000	1.000	000’

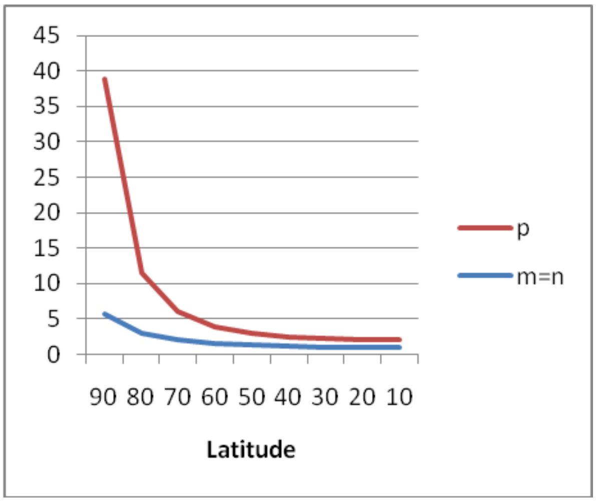

Fig. 3. The diagram of relative linear distortion in Mercator projection

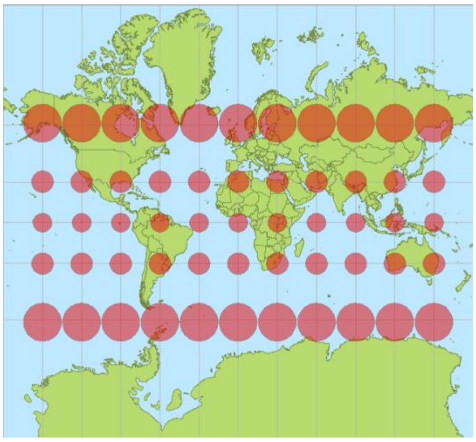

Fig. 4. The world map and the distribution of the distortions

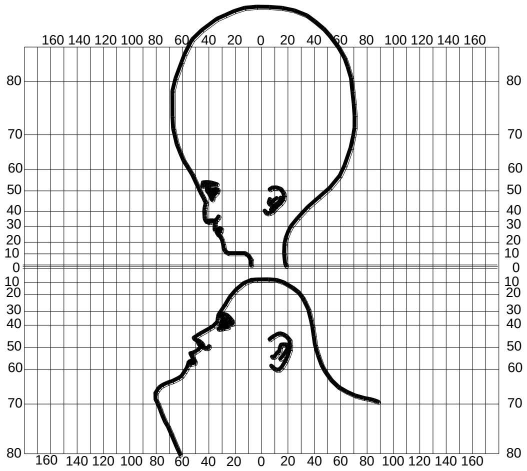

Fig. 5. Distribution of the distortions using the human profile

It is noticed that while the angles are still not deformed on whole surface of the map, not the same thing can be said about the distances and surfaces. So, the distances are the more deformed the more they are distanced from the line of null distortion, which is in this case the Ecuador. As an example, at the latitude of 600, the distances are represented into the projection twice bigger at the Ecuador, where the scale is equal to the main scale. The surfaces are more deformed than the distances

The maps in this projection have a great importance in maritime and air navigation, because the projection is conform and the cartographic network formed in perpendicular lines, the loxodrome (rhumb line) will be a straight line. This line makes with each meridian projection the same azimuth.

This important feature of the maps in Mercator projection constitutes a facility for leading the planes and ships regarding their commodity during their trips.

Indeed, on a map in Mercator projection, the straight line AB represents the loxodrome, and the curve line AB orthodrome (great circle - that means the shorter distance between 2 points on the globe and that represents the arc of the big circle that crosses through A and B).

Going from B to A and using a map in Mercator projection, the commander of the ship takes care that the direction of going forward to make with the meridian projection the same angle α.

For this, he will have to go after the loxodrome AB that on the mercatoriane map is projected after the straight line AB, but which, on the glob, has a curve way, so, it is much bigger that the straight line – the oxodroma, the shorter line that joins 2 points on the globe and that is an arc of the big circle that crosses through these 2 points – the orthodromethat in Mercator projection is represented by a curve line AB.

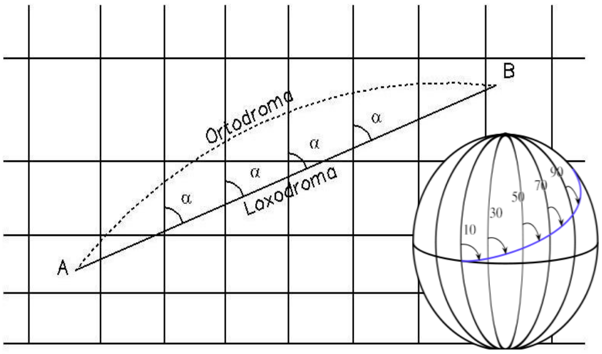

Fig. 6. Loxodrome (rhumb line) and orthodrome in Mercator projection

When the ship follows the loxodrome it will make an indirect way, so bigger than the shortest distance between the 2 points on the globe – the orthodrome -, so, in this way it has the advantage to know, in any point, the direction of orientation of the ship and so, to keep its direction.

For example: the distance between Moscow – San Francisco after the orthodrome– the arc of the big circle that crosses through these 2 places – is of 9476 km, and the loxodrome in Mercator projection, even if it is represented in these maps with a straight line, it has a length of 10051 km, so with 575 km longer than the orthodrome.

Loxodrome and problems of loxodrome. It is considered on the globe the distance AB that must be made by a ship between the towns A and B, situated on different meridians and parallels (Fig.7).

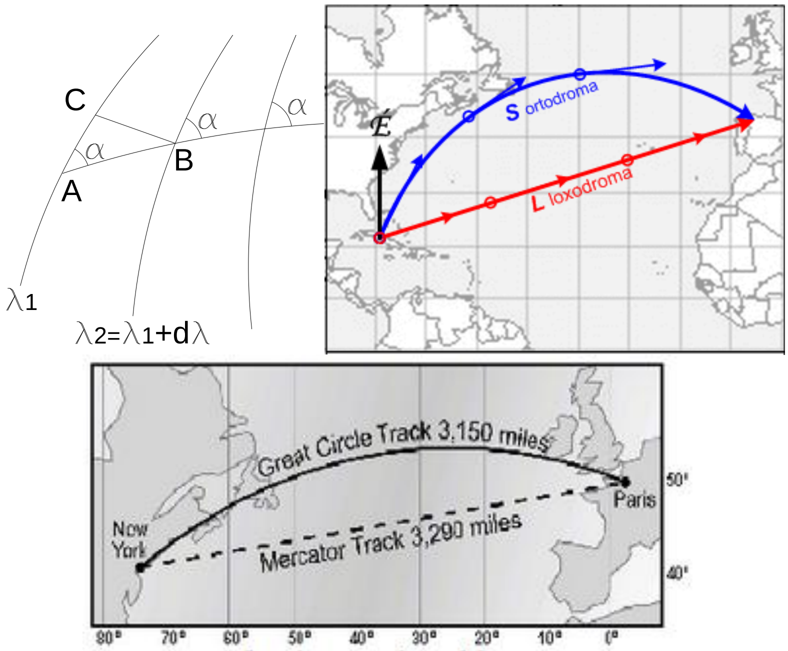

Fig. 7.The representation of loxodrome (rhumb line)

From the triangle ABC results:

$$tg\,\alpha = \frac{BC}{AC} = \frac{N\cos\alpha\, d\,\lambda}{M\, d\,\varphi}$$

$$M\, d\,\varphi \cdot tg\,\alpha = N\cos\alpha\, d\,\lambda$$

$$d\,\lambda = \frac{M\, d\,\varphi}{N\cos\alpha}\cdot tg\,\alpha \qquad (9)$$

Integrating it we shall obtain:

$$\int_{\lambda_1}^{\lambda_2} d\,\lambda = tg\,\alpha \int_{\lambda_1}^{\lambda_2} \frac{M\, d\,\varphi}{N\cos\varphi}$$

$$\lambda_1 - \lambda_2 = tg\,\alpha\left(\log U_2 - \log U_1\right) \qquad (10)$$

Assuming that into the starting point of the ship A(φA, λA) taking into consideration φ1 = 0, λ1 = 0, we shall have:

$$\lambda = tg\,\alpha \cdot \log U \qquad (11)$$

This expression represents the equation of the loxodrome on the revolution ellipsoid; in case when the earth is considered a sphere, the equation of the loxodrome will have the following form:

$$\lambda = tg\,\alpha \cdot \log tg\left(45^0 + \frac{\varphi}{2}\right) \qquad (12)$$

For φ = 00, we have U = ∞; it results that the loxodrome is a spiral that surrounds the 2 points from the pole that constitutes the asymptotic point.

Near the pole it is situated the logarithmical spiral due to the following fact:

$$\int \frac{d\,\varphi}{\cos\varphi} = -\int \frac{d\,\lambda}{\cos\lambda} = -\int \frac{d\,\lambda}{\lambda} = -\log\lambda + K \qquad (13)$$

Into the projection plan the loxodrome has the aspect of a straight line that intersects the meridians projections under the same angle (Fig. 8).

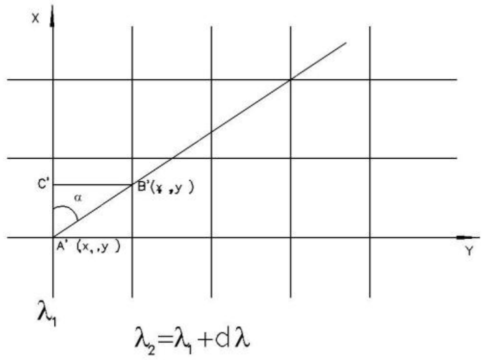

Fig. 8. Determination of the loxodrome

If we consider into the projection plan the loxodrome between the coordinates points A′(x1, y1) and B′(x2, y2); from the triangle A′B′C′, results:

$$tg\,\alpha = \frac{B'C'}{A'C'} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\lambda_2 - \lambda_1}{D_2 - D_1} \qquad (14)$$

From these relations, it results that the loxodrome into the Mercator projection is represented into the projection plan as a straight line; it does not show the shorter distance between the 2 points on the globe. So, the orthodrome in this projection is represented with a certain curve.

- **UTM PROJECTION**

This is the most common planar representation of the earth. Commonly used for medium and large scale topographic maps. The system provides spatial coordinates at high levels of precision for most of the globe. It was established in 1936 and adopted by the US army in 1947. This projection, which is a variant of the *Gauss – Krüger* projection, it is important also for Romania after our integration in new political and military structure.

Into the UTM system (Universal Transversal Mercator) the projection of the curved/spherical surface of the earth can be made on a cylinder whose axis is perpendicular on the poles axis.

This arrangement makes impossible to be represented the whole surface of the earth on the same plan, the projection being made on different plans, each one along one meridian called the central meridian. In order to minimize the distortions it was chose that the height of a part (called spindle) should be of 60 as longitudinal into an interval limited of the parallel of 800 south latitude and 840north latitude, resulting 360° / 6° = 60 areas (spindles).

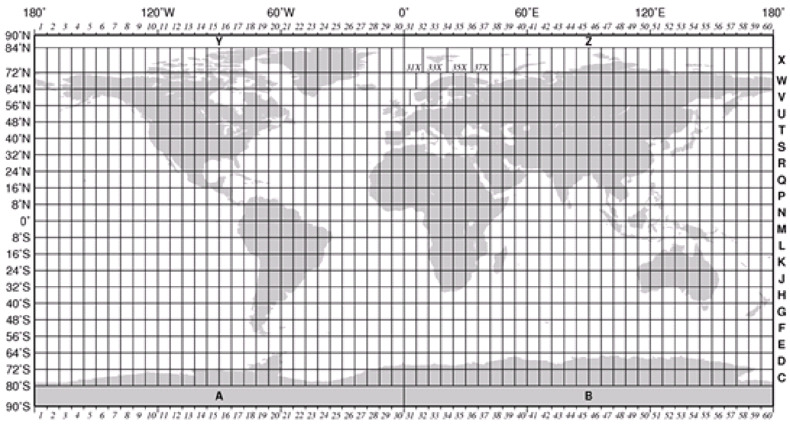

Fig. 9. The map of World in UTM projection

Numbering the spindles starts from the meridian of 180° from the Pacific Ocean. Romania is partial into the spindle 34 (18°-24°) (its western half) and partial into 35 (24°-30°) (its eastern half).

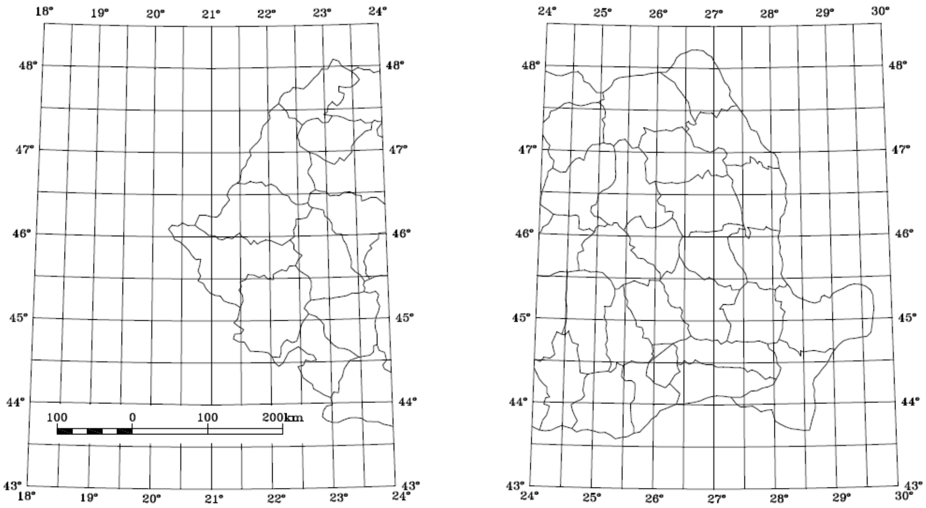

Fig. 10. The map of Romania in UTM projection

Into each spindle there is a central point of the projection at the intersection between the Ecuador and the central meridian of the projection. For the spindle 34 the central meridian of the projection is 210 and for the spindle 35 the meridian 27°. Normally the origin of the system should be this point of intersection (x = 0, y = 0). But, this would mean that at the left from the origin we should have negative values and lower from the origin also. Due to this fact it was made an artifice for eliminating the negative values: on the abscise the system origin (zero point) is placed with 500 km more to west (left), so the whole space represented inside the same spindle should enter into the positive values area.

The reference ellipsoid is the international ellipsoid called WGS – *84,* for which:

Great semi ax: $a = 6378137{,}000\ m$

Geometrical flattening: $f = 1/298.257223563$

As basic scales the NATO standards proposes the scales of 1:250 000 and 1:50 000. each scale has its own nomenclature.

- ***Map sheet nomenclature at scale of 1:250 000***

Dividing the ellipsoid surface in fuses corresponds with the draft for the international map sheets where the Earth surface is covered with a series of geometrical figures whose dimensions are of 6° longitude and 4° latitude. The fuses of 6° longitude in Mercator projection are numbered from 1 to 60 starting with the meridian of 180°, the areas of 4° latitude are numbered with big letters of the Latin alphabet from A to V starting with Equator.

The map at scale of 1:1 000 000 was taken as base for the maps of scale 1:250000. It results that for obtaining a map sheet at scale of 1 : 250 000 the map sheet of 1:1 000 000 was divided from 1° to 1° on latitude and from 2° to 2° on longitude that leads to obtaining 12 map sheets at scale of 1:250 000 noted with Arabian numbers from 1 to 12.

- ***Map sheet nomenclature at scale of 1:50 000***

Map sheet nomenclature at scale of 1:50 000 is based on the map at scale of 1:1 000 000. The dimensions of the sheet in Romania are 15° on latitude and 18° on longitude. For obtaining a map sheet at scale of 1:50 000 000 the map sheet of 1:100 000 is divided in 4 from 30° to 30° latitude and from 36° to 36° on longitude. The nomenclature of the map sheet is composed of groups of alpha-numerical characters separated by the symbol "x".

The first group of characters is composed of a letter and three figures having the following significance:

The first character (letter) means the region of shore delimited by the NATO interest (Central Europe and South – East Europe). Romania belongs to the M region.

The second character is a figure which shows that the map has a certain scale. This character has a value between 0 – 9 depending on the scale with the following values:

1 for the scales less than 1: 5 000 000
2 for scales between 1 : 2 000 000 + 1:5 000 000
3 for scales between 1: 510 000 + 1:2 000 000
4 for scales between 1: 255 000 +1:510 000
5 for scales between 1:150 000 * 1:255 000
6 for scales between 1; 70 000 + 1: 150 000
7 for scales between 1: 35 000 ^ 1: 70 000
8 for scale bigger than 1 :35 000 excepting the town plans
9 town plans
10 photo maps

The third character is a geographical area from the given region. For Romania, Greece, Bulgaria and ex Yugoslavia is the figure 0.

The forth character is a given sub-area. For Romania is the figure 5.

The second group of characters is formed of 3 groups of figures:

The first group is formed of 2 figures representing the numbering of the areas of 36° on longitude from east to west.

The second group is formed of 2 figures representing the numbering of the areas of 30° on latitude from south to north

The third group is formed from a figure which represents the position of the map sheet inside the quadrilater.

The **relative linear distortion** has the following formulas:

$$D_{UTM} = k(D_{Gauss} + 1) - 1 = k(L^2 / 2R^2 + L^4 / 24R^4 + 1) - 1 \ \text{[km/km]} \qquad (15)$$

where:
DUTM relative linear distortion in UTM projection;
DGauss relative linear distortion in Gauss projection;
R is the average ray of flexion into the considered point;
y=(y-y 0) is distance between the point and axial meridian;
k is the constant report between the distances from the UTM projection plan and the ones from the Gauss projection plan.

By using this formula for the relative linear distortion in UTM projection there are obtained values which are directly proportional with the distance against the axial meridian and they increase starting with the negative value -40 cm/km (Fig. 11).

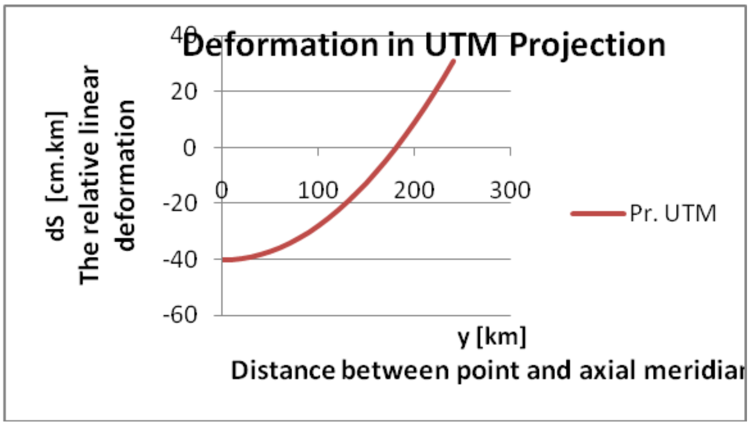

Fig. 12. The diagram of relative linear distortion in UTM projection

RESULTS AND DISCUSSIONS

➢ **Mercator projection**

- **Advantages**

The practical importance of the Mercator projection consists in the fact that it has all the qualities of a map used in maritime navigation.

A maritime map must fulfill the following conditions:

It should be fixed easily the position of a point by using its coordinates and respectively it should be determined the coordinates of a point. For an easy solving of this thing it is good that the meridians and the parallels should be perpendicular.

The map should be built in a conform projection.

The loxodrome should be represented by a straight line.

The distances should be measured easily on it.

The first 2 conditions are fulfilled by the constructing the projection, and the third one is done by the projection being conformal and having the meridians as straight parallel lines, resulting fact being that the loxodrome can be represented by straight line.

This projection is used for drawing navigation maps, or the planispher in which case the cylinder is tangent to the sphere, or for maps of the ocean basins where the cylinder is secant.

- **Disadvantages**

The Mercator projections deforms radical the measures of the north latitude making larger Greenland and Russia, for example and making thinner Australia and Africa.

For example, the biggest island of the world, Groelanda, has an aria almost of 2.175.600 km2. But on the Mercator map it appears bigger than South America, having an area of almost 17.835.000 km2 on the Ecuator.

The centering of the map on Eurasia gives inevitable this region as being pivotal and America as margins.

The Mercator projection, due to its distortions on the latitude, imposes a specific method for determining the real distance, when it not noted the scale of Mercator map at average latitude of the respective plans.

- **UTM projection**

- **Advantages**

The UTM system that uses the Mercator projection can be used on the whole earth globe having the advantage that it reduces the errors for representing in plan due to introducing a factor of scale, which makes that the linear distortions from the margin of the spindle projected into plan should be reduced at half.

By adopting the representation system on spindles of 60 longitudes, the representation in plan is almost true.

By using this projection which, being a conformal projection because it does not deform the angles and the module of linear distortion, being small, leads to a precise representation of the whole globe.

- **Disadvantages**

A full reference requires a zone number and easting and northing.

The axes in adjacent zones are skewed; therefore problems arise when working across zone boundaries.

There is no mathematical relationship between coordinates in one zone and those in an adjacent zone.

BIBLIOGRAFY

1. HERBEI O. & HERBEI M. – "Geographic Information Systems. Theoretical And Applications", Universitas Press, Petrosani, 2010;
2. BONEA, I., DIMA, N., Mathematical Cartography – Making And Editing The Maps, Lit. I.M.P., Petrosani, 1970
3. DIMA, N., Geodesy, Universitas Publishing House, Petrosani, 2005
4. HERBEI, O., Mathematical Cartography – Making And Editing The Maps, Eurobit Publishing House, Timisoara, 2002
5. PALAMARIU, M., PĂDURE, I., ORTELECAN M., Cartography And Cartonometry , Aeternitas Publishing House, Alba Iulia, 2002